



Mahidol  
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*Wisdom of the Land*



# Estimation of the basic reproduction number ( $R_0$ )

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# Estimation of $R_0$

1. Endemic equilibrium
2. Epidemic episode

$R_0$  estimation

$\beta$  estimation

-> ( $R_t$ ) Estimation



# Estimation of $R_0$

## 1. Endemic equilibrium

If homogenous population, born susceptible, no lost of immunity,...

$$R_0 = \frac{L}{A}$$

← Average life expectancy  
← Average infection age

Average life expectancy 80 years and Average infection age 10 years,  $R_0=8$

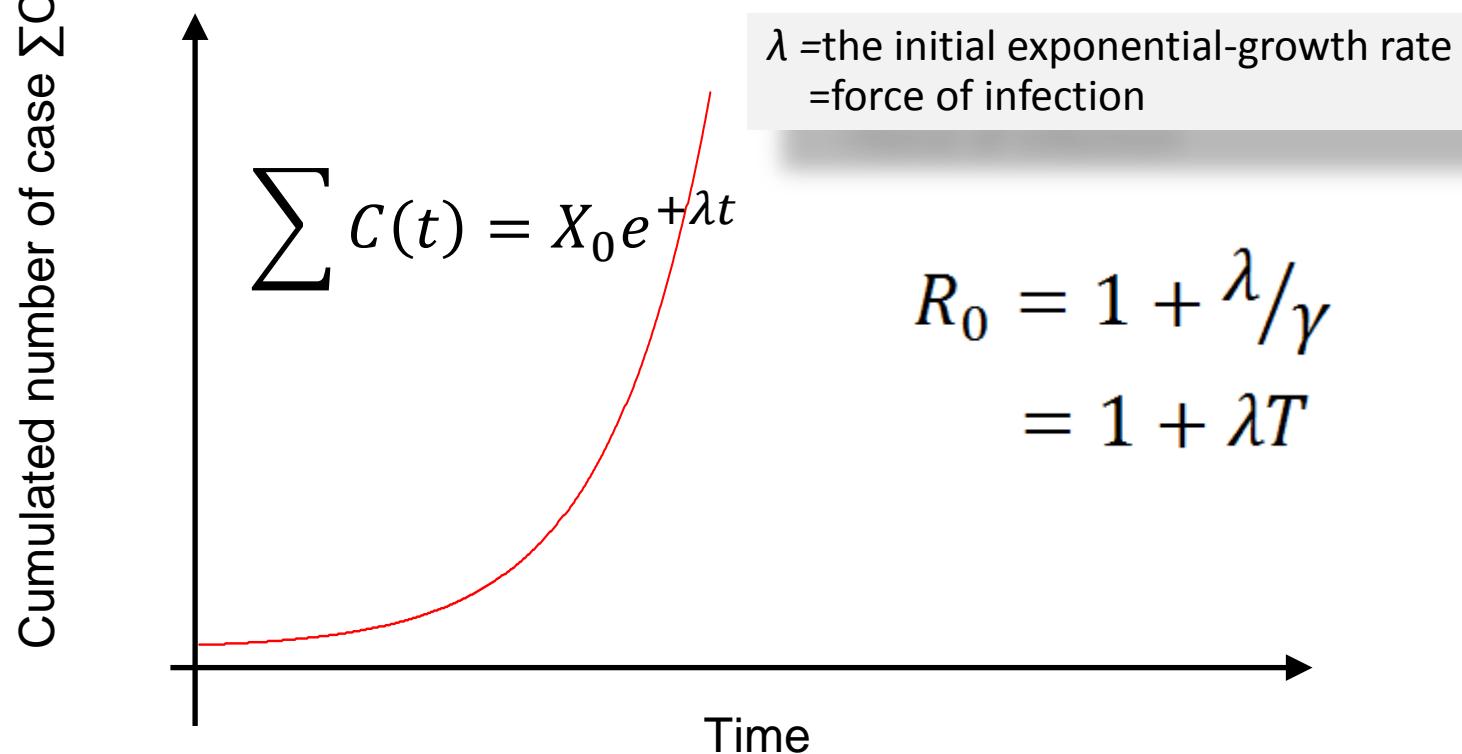
$$\lambda = \frac{1}{A}$$

Force of infection

# Estimation of R<sub>0</sub>

## 1. Epidemic episode

R<sub>0</sub> estimation : exponential growth

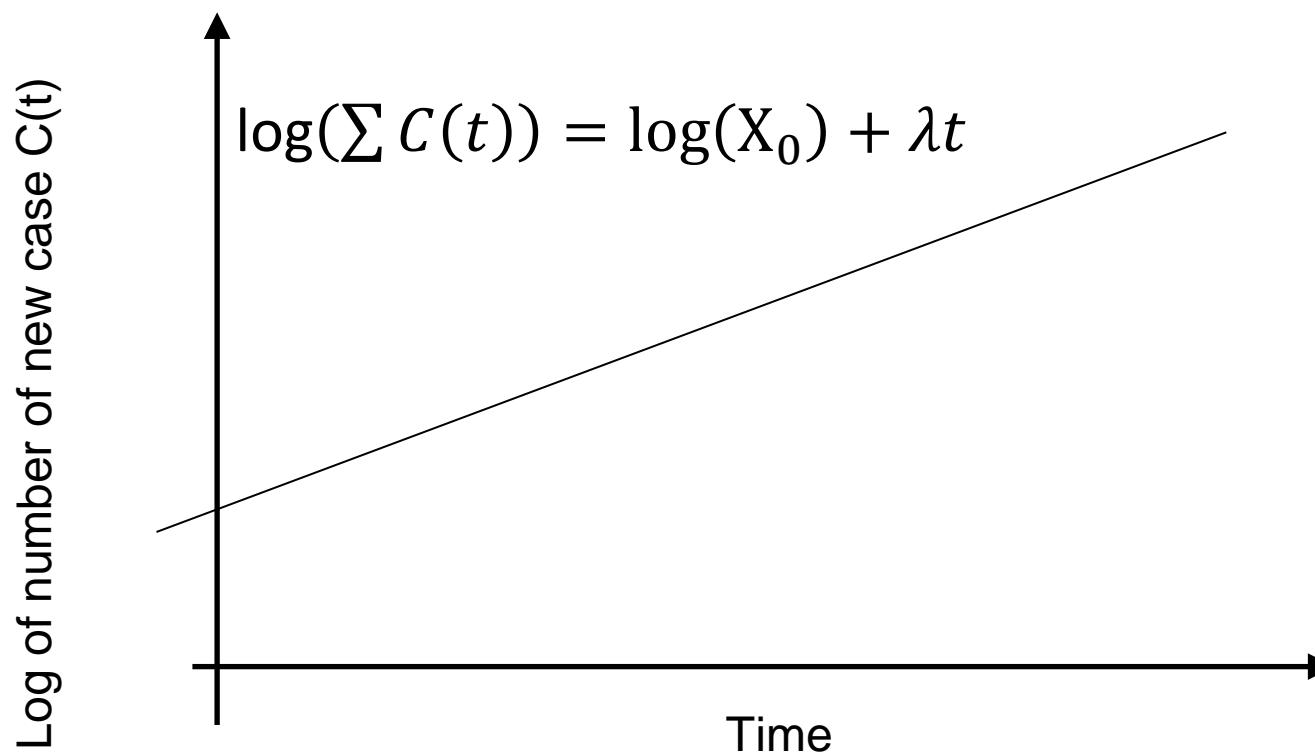




# Estimation of $R_0$

## 1. Epidemic episode

$R_0$  estimation : exponential growth

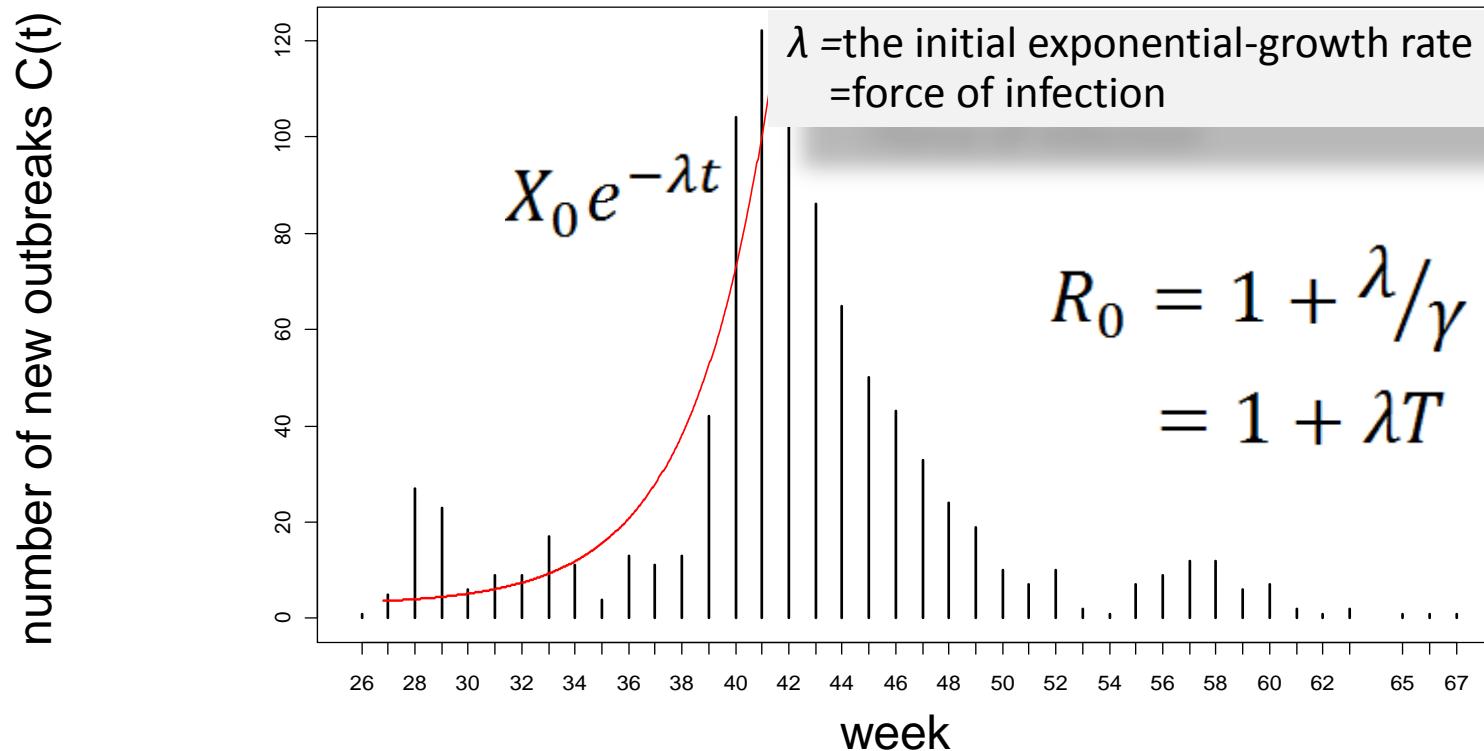




# Estimation of $R_0$

## 1. Epidemic episode

$R_0$  estimation : exponential growth



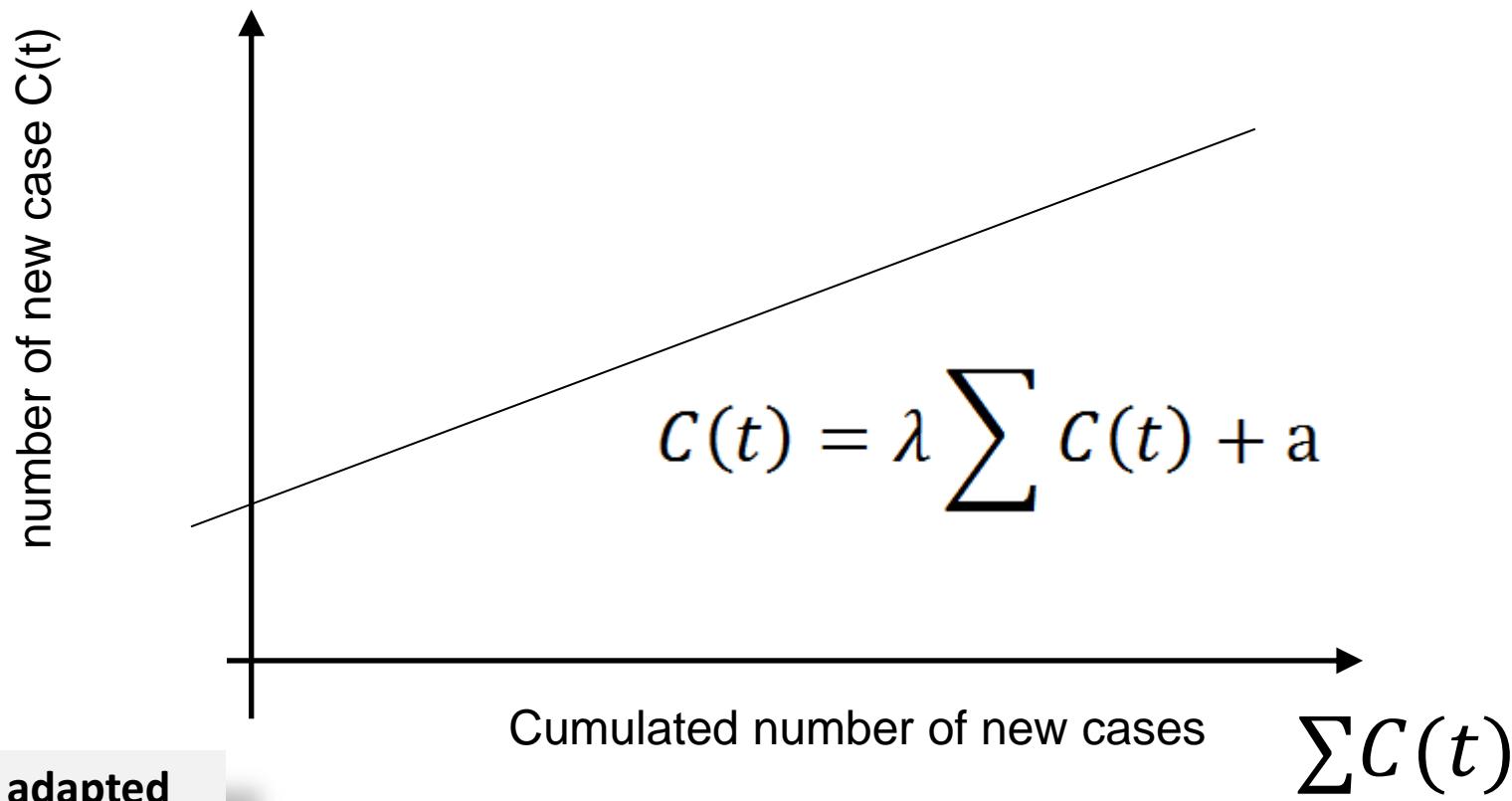
- 1. Only at the beginning
- 2. A global value
- 3. Not easy to define the limits of the phase





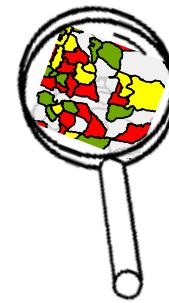
# Estimation of $R_0$

## $R_0$ estimation : exponential growth

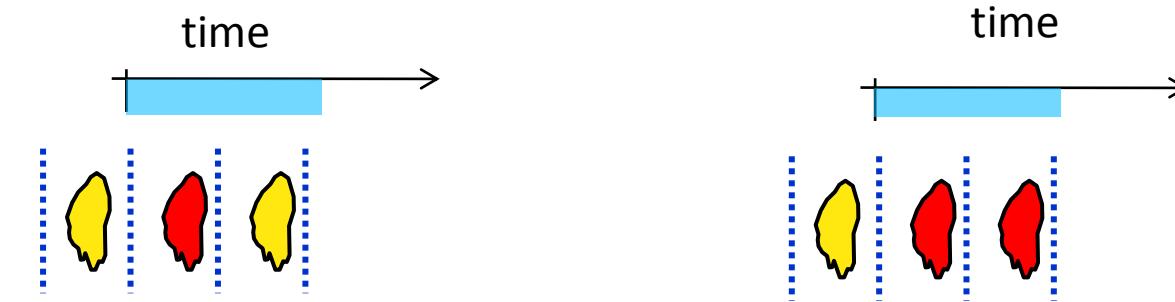


Method adapted  
from (Favier et al.,  
2006)

# Estimated basic reproduction rate



$R_0$



	2 week refractory period and 1 week infectious period (T)	2 week refractory period of and 2 week infectious period (T)		
	Method adapted from <a href="#">(Vynnycky et al., 2007)</a>	Method adapted from <a href="#">(Favier et al., 2006)</a>	Method adapted from <a href="#">(Vynnycky et al., 2007)</a>	Method adapted from <a href="#">(Favier et al., 2006)</a>
Initial exponential-growth rate (SE)	$\lambda=0.267$ (SE=0.045)	$\lambda=0.310$ (SE=0.038)	$\lambda=0.262$ (SE=0.044)	$\lambda=0.299$ (SE=0.039)
$R_0$ [95% CI]	$R_0=1.267$ [1.180-1.355]	$R_0=1.310$ [1.234-1.385]	$R_0=1.526$ [1.352-1.670]	$R_0=1.597$ [1.444-1,750]

# Estimation of $R_0$



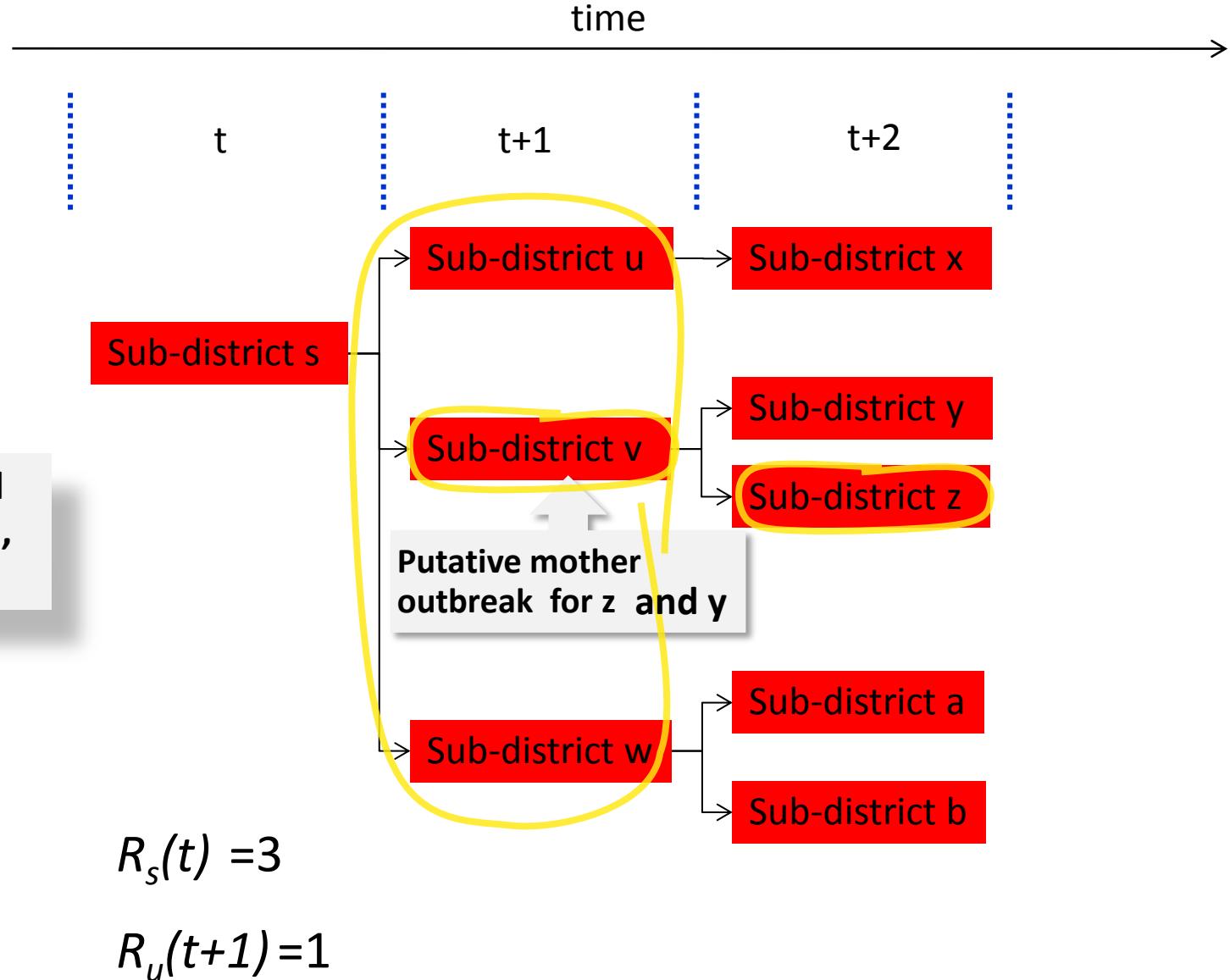
$R_0$  estimation : exponential growth

Nearest infectious neighbor method

-> Epidemic tree

# $R_0$ estimation : exponential growth

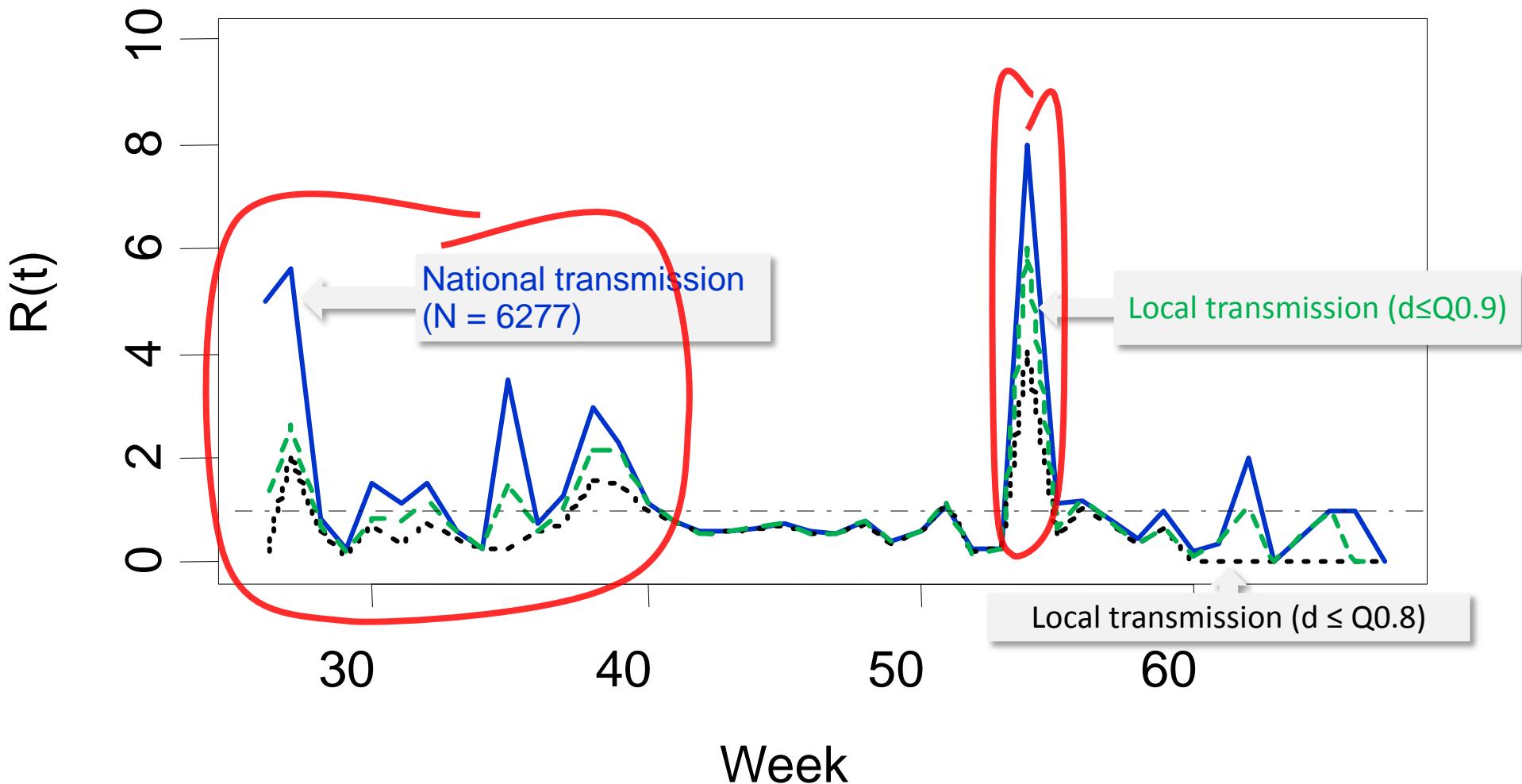
## Epidemic tree using the nearest infectious neighbor method



# Estimations of the R(t)



## 2. Application to the estimation



# Effective reproduction number $R(t)$



- When the population cannot be considered as being completely susceptible
- It represents the expected number of secondary cases issued from a primary case.

$$R \leq R_0$$

$$R = \beta T$$

- > Estimation de  $\beta$
- > Estimation de  $T \Rightarrow$  Survival curve

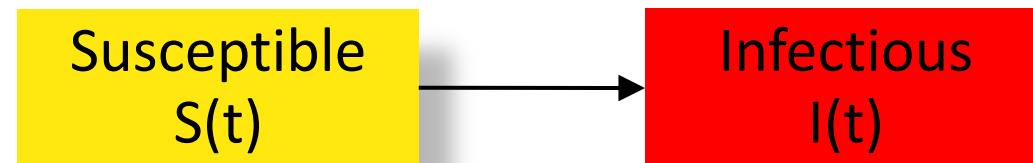


# Effective reproduction number R



-> Estimation de  $\beta$

Method adapted  
from (Ward et al.,  
2009)



$$N = S(t) + I(t)$$

$$\beta(t)S(t)I(t)/N = C(t)$$

$$R(t) = \beta(t)T$$



# Effective reproduction number R



-> Estimation de  $\beta(t)$

Method adapted from  
(Eblé et al., 2008)

$$\beta(t)S(t)I(t)/N = C(t)$$

$$P(\text{not inf}) = e^{-\beta I \Delta t / N}$$

$$P(\text{inf}) = 1 - e^{-\beta I \Delta t / N}$$

$$C(t) = S(t) * (1 - e^{-\beta I \Delta t / N})$$

$$1 - C(t)/S(t) = e^{-\beta I \Delta t / N}$$

$$\log(1 - C(t)/S(t)) = -\beta I \Delta t / N$$

$$\log\left(-\log\left(1 - \frac{C(t)}{S(t)}\right)\right) = \log(\beta) + \log(I \Delta t / N)$$

# Estimation of R



- > Various methods
- > According data
- > According objective
- > different values and  
different precisions
  
- > Let us practice